

V SEM(CBCS) MODEL QUESTION PAPER – B. Sc. MATHEMATICS
PAPER: 5.1: ADVANCED ALGEBRA AND NUMERICAL METHODS

Time : 3 Hours

Max. Marks : 90

PART-A

I. ANSWER ANY SIX QUESTIONS.

6X2=12

1. Add and Multiply the two polynomials over the Ring $(\mathbb{Z}_6, +_6, \times_6)$, where $f(x)=2x^0+5x+3x^2$ and $g(x)=1x^0+4x+2x^3$.
2. Define Irreducible element ?
3. Give an example to show that the union of two sub rings is not necessarily a Sub-ring .
4. Express the vector $(3,5,2)$ as the linear combination of the vectors $(1,1,0)$, $(2,3,0)$ and $(0,0,1)$ of $V_3(\mathbb{R})$.
5. Find the Linear transformation of $f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ such that $f(1,0)=(1,1)$ and $f(0,1)=(-1,2)$.
6. Explain Newton-Raphson Method to solve $f(x)=0$.
7. Solve $\frac{dy}{dx} = x + y$ by Euler's Method, given that $y=1$ when $x=0$, $h=0.2$.

PART-B

II. ANSWER ANY SIX QUESTIONS.

6X3=18.

1. Show that the set M of all 2×2 matrices of the form $\begin{bmatrix} 0 & a \\ 0 & b \end{bmatrix}$ where a, b are integers, is a left ideal but not a right ideal in the right of 2×2 matrices with elements as Integers.
2. If R is a commutative Ring, then
 - (i) $a|b$ and $b|c \Rightarrow a|b+c$.
 - (ii) $a|b \Rightarrow a|bx$ for all $x \in R$.
3. Let $f: R \rightarrow R^1$ be a homomorphism of rings from R onto R^1 with kernel K , then show that f is one-one iff $K = \{0\}$.
4. In any Vector Space V over a field F . Prove that
 - (i) $c \cdot 0 = 0, \forall c \in F$
 - (ii) $0 \cdot \alpha = 0, \forall \alpha \in F$
 - (iii) $(-c)\alpha = -(c\alpha) = c(-\alpha), \forall c \in F \& \forall \alpha \in F$
5. For the Matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, find the corresponding Linear Transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ w.r.t the Basis $\{(1,0), (1,1)\}$.
6. Use Gauss-Seidel Method to solve the system $x + 5y = 11; 4x + y = 6$ correct to 2 decimal places.
7. Using Picard's method, solve the differential equation $\frac{dy}{dx} = x^2 + y, y(0)=1$ to find the value of y at $x=0.1$.

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PART -C

III. ANSWER ANY FOUR QUESTIONS

4x5=20.

1. Prove that the Ring $(\mathbb{Z}_n, +_n, \times_n)$ is an Integral Domain and hence a Field if and only if n is a prime.
2. The necessary and sufficient conditions for a non-empty subset K of a field F to be subfield of F are
 - (i) $a \in K, b \in K \Rightarrow a-b \in K$.
 - (ii) $a \in K, 0 \neq b \in K \Rightarrow ab^{-1} \in K$.
3. Show that the union of two ideals of a ring R is an ideal of R if and only if one is contained in the other.
4. Let $f: R \rightarrow R^1$ be a homomorphism from the Ring R into R^1 then prove that
 - (i) $f(0) = 0^1$ are the zeros of R and R^1 respectively.
 - (ii) $f(-a) = -f(a), \forall a \in R$.
5. State and Prove the Fundamental theorem of homomorphism.

IV. ANSWER ANY FOUR QUESTIONS.

4x5=20.

6. Prove that the union of two subspaces of a vector space V over a field F is a subspace if and only if one is contained in the other.
7. In a n -dimensional vector space $V(F)$, (i) any $(n+1)$ elements of V are Linearly Dependent.
(ii) no set $(n-1)$ elements can span V .
8. If T is a mapping from $V_2(R)$ into $V_3(R)$ defined by
 $T(x,y) = (x \cos \theta - y \sin \theta, x \sin \theta + y \cos \theta)$, then show that T is a Linear Transformation.
9. Let $T: V \rightarrow W$ be a linear transformation and V be a finite dimensional vector space, then prove that $r(T) + n(T) = d(V)$.
10. Show that the linear map $T: V_3 \rightarrow V_3$ defined by $T(e_1) = e_1 + e_2, T(e_2) = e_2 + e_3$ and $T(e_3) = e_1 + e_2 + e_3$ is non-singular and find its inverse.

V. ANSWER ANY FOUR QUESTIONS.

4x5=20.

11. Find the root between 2 and 3 of the equation $x^4 - x^3 - 2x^2 - 6x - 4 = 0$ up to six iterations correct to two decimal places by using Bisection Method.
12. The equation $x^3 - 4x + 1 = 0$ has one root between 0 and 1. Find the real root correct to 3 places of decimal by the Method of False Position.
13. Solve by Jacobi's Iteration Method $5x + 2y + z = 12, x + 4y + 2z = 15, x + 2y + 5z = 20$.
14. Using Euler's Modified Method, Solve $\frac{dy}{dx} = x^2 + y$ when $y(0)=1$ for $x = 0.25$.
15. Solve $\frac{dy}{dx} = x + y^2$ with the initial condition $y=1$ when $x=0$ for $x = 0.2$ by Runge - Kutta Method.

Method.

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V SEM(CBCS) MODEL QUESTION PAPER – B. Sc. MATHEMATICS

PAPER: 5.1: ADVANCED ALGEBRA AND NUMERICAL METHODS

Duration: 3 hrs

Max. Marks: 90

PART-A

I. ANSWER ANY 6 QUESTIONS

6X2=12

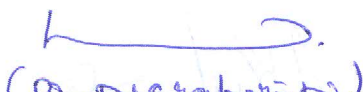
1. Give an example of the following: (i) A ring with zero divisors (ii) Field.
2. Define an ideal of a ring.
3. State the fundamental theorem of homomorphism of rings.
4. Define a subspace of a vector space. Give an example.
5. Find the rank of the linear transformation whose matrix is $M = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 1 \\ 2 & 2 & -1 \end{bmatrix}$.
6. State the intermediate value theorem.
7. Using Picard's method, solve the differential equation $\frac{dy}{dx} = x + y$, $y(0) = 1$ to find the value of y at $x = 0.2$ correct to two decimal places.

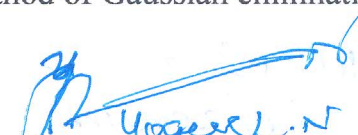
PART-B


II. ANSWER ANY 6 QUESTIONS

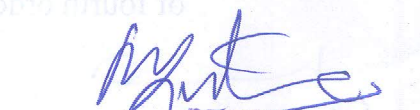
6X3=18

1. Prove that in a ring $(R, +, \cdot)$, $a \cdot 0 = 0 \cdot a = 0 \quad \forall a \in R$ where 0 is the additive identity of R .
2. Prove that $S = \left\{ \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} : a \in \mathbb{Z} \right\}$ is a sub-ring of the ring of integers $(\mathbb{Z}, +, \cdot)$.
3. Define the following w. r. to a ring: (i) Unit elements (ii) Irreducible elements.
4. In a vector space $(V, +, \cdot)$ over the field F , show that $c \cdot (-\alpha) = (-c) \cdot \alpha = -(c\alpha) \quad \forall c \in F, \alpha \in V$.
5. Define basis and dimension of a vector space.
6. Apply Regula-Falsi method to find a root of the equation $x^2 - 3x - 1 = 0$ in the interval $[3, 4]$ in two steps.
7. Use the method of Gaussian elimination to solve the system $x - y = 5$, $2x + y = 4$.


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PART-C

III. ANSWER ANY 4 QUESTIONS

4X5=20

1. Show that the set $Z[i]$ is a Ring under usual addition and multiplication of complex numbers.
2. Prove that the ring $(Z_n, +_n, \cdot_n)$ is an integral domain if and only if n is prime.
3. Show that the intersection of any two subrings of a ring R is again a subring of R . Further, give an example to show that the union of two subrings need not be a subring.
4. Prove that the ring of integers $(Z, +, \cdot)$ is a principal ideal ring.
5. a) Define the kernel of a ring homomorphism.
b) Prove that $f : (Z, +, \cdot) \rightarrow (3Z, +, \cdot)$ defined by $f(x) = 3x \forall x \in Z$ is an isomorphism.

IV. ANSWER ANY 4 QUESTIONS

4X5=20

6. Show that the set $V_2(R) = \{(x, y) \mid x, y \in R\}$ forms a vector space over the field R w. r. to component-wise addition and scalar multiplication.
7. Show that the vector $(2, -5, 7) \in V_3(R)$ is not in $L[S]$, where $S = \{(1, -3, 2), (2, -4, -1), (1, -5, 7)\}$.
8. Define linear independence and dependence of a set of vectors. Further, prove that, in a vector space of dimension n , any set of vectors $\{v_1, v_2, \dots, v_n, v_{n+1}\}$ is linearly dependent.
9. Show that $T : V_2(R) \rightarrow V_2(R)$ defined by $T(x, y) = (x + y, x - 2y)$ is a linear transformation. Also, find the matrix of the transformation w. r. to the standard ordered basis of $V_2(R)$.
10. Define range and kernel of a linear transformation. Further, if $T : U \rightarrow V$ is a linear transformation, prove that $\text{range}(T)$ is a subspace of V .

V. ANSWER ANY 4 QUESTIONS

4X5=20

11. Use the method of bisection to obtain a real root of the equation $xe^x = \cos x$ correct to two decimal places.
12. Obtain a root of the equation $x \cos x - \sin x = 0$ near $x = 0.5$ using Newton Raphson method.
13. Solve the system of equations $x + 10y + z = 24$, $x + y + 10z = 33$, $10x + y + z = 15$ using Gauss-Seidel method taking 4 iterations.
14. Using Euler's modified method, solve $\frac{dy}{dx} + y = 1$, $y(0) = 0$ for $x = 0.2$.
15. Find the value of y at $x = 1.25$ given that $\frac{dy}{dx} = xy$, $y(1) = 2$ using Runge Kutta method of fourth order.

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Set-1

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V SEM(CBCS) MODEL QUESTION PAPER – B. Sc. MATHEMATICS

PAPER – 5.2a: Analysis and Integral Transforms

Duration : 3 Hours

Max. Marks : 90

PART- A

I Answer any Six Questions.

6 × 2 = 12

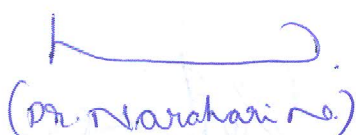
- 1) Prove that $\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$
- 2) Prove that $\Gamma(1/4)\Gamma(3/4) = \pi\sqrt{2}$.
- 3) If $f(x) = \begin{cases} 0 & \text{for } -2 < x < 0 \\ 1 & \text{for } 0 < x < 2 \end{cases}$ then find b_n .
- 4) Find $L[\sin 3t \cos 2t]$
- 5) Find $L\left[\frac{e^{-t} \sin t}{t}\right]$
- 6) Prove that $F[af(t) + bg(t)] = aF[f(t)] + bF[g(t)]$.
- 7) Write down the Fourier Cosine and Sine Transforms of $f(x)$.


PART -B

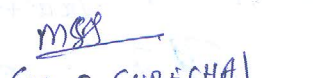
II Answer any Six Questions

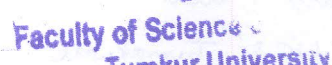
6 × 3 = 18

- 1) Show that $\int_0^a x^4 \sqrt{a^2 - x^2} dx = \frac{\pi a^6}{32}$
- 2) Evaluate $\int_0^{\infty} \frac{x^4(1+x^5)}{(1+x)^{15}} dx$
- 3) Obtain the Fourier series of $f(x) = \frac{1}{2}(\pi - x)$ in $-\pi < x < \pi$
- 4) Find the $L[\cos ht \sin^3 2t]$
- 5) Find $L^{-1}\left[\frac{2s-1}{s^2-2s+10}\right]$
- 6) Find the Fourier Cosine Transforms of $f(x) = \begin{cases} 1 & \text{for } 0 \leq x < a \\ 0 & \text{for } x \geq a \end{cases}$
- 7) Find the Fourier Integral Expansion of $f(x) = \begin{cases} \pi/2 & \text{for } |x| < 1 \\ 0 & \text{for } |x| > 1 \end{cases}$


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PART- C

III Answer any Four Questions

$4 \times 5 = 20$

- 1) Prove that $\int_0^{\pi/2} \sin^m \theta \cos^n \theta d\theta = \frac{1}{2} \left(\frac{\Gamma(m+1/2)\Gamma(n+1/2)}{\Gamma(m+n+1/2)} \right)$
- 2) Show that $\int_0^1 \frac{dx}{(1-x^n)^{1/n}} = \frac{\pi}{n} \operatorname{cosec} \left(\frac{\pi}{n} \right)$
- 3) Obtain the Fourier Series of $f(x) = \begin{cases} -\pi & \text{in } -\pi < x < 0 \\ x & \text{in } 0 < x < \pi \end{cases}$
and Hence find that $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots$
- 4) Obtain the Fourier Series of $f(x) = \begin{cases} 1 + \frac{4x}{3} & \text{for } -3/2 < x \leq a \\ 1 - \frac{4x}{3} & \text{for } 0 \leq x < 3/2 \end{cases}$
- 5) Obtain half range cosine series of $f(x) = x(\pi - x)$ over the interval $(0, \pi)$
and hence deduce that $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$

IV Answer any Four Questions:

$4 \times 5 = 20$

- 6) Evaluate (a) $L[e^{-3t}(2\cos 5t - 3\sin 5t)]$ (b) $L(1+t)^3$
- 7) Evaluate (a) $L \int_0^t t e^{-t} \sin at dt$ (b) $L \left[\frac{e^{-t} \sin t}{t} \right]$
- 8) Find the $L^{-1} \left[\frac{s}{(s-3)(s^2+4)} \right]$
- 9) Verify the Convolution theorem for $f(x) = e^t, g(t) = \cos t$
- 10) Solve by using Laplace Transforms $\frac{d^2 y}{dt^2} - 2 \frac{dy}{dt} + y = e^{2t}$ given that $y = 1, \frac{dy}{dt} = -1$ when $t = 0$.

V Answer any Four Questions

$4 \times 5 = 20$

- 11) Find the Fourier integral Expansion of $f(x) = \begin{cases} 0 & \text{in } x < 2 \\ 1 & \text{in } 2 < x < 3 \\ 0 & \text{in } x > 3 \end{cases}$
- 12) Find the Fourier Transform of $f(x) = \begin{cases} 1 - |x| & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ and hence Show
that $\int_0^{\infty} \frac{\sin^2 t}{t^2} dt = \frac{\pi}{2}$
- 13) Find the inverse Fourier transform of $f(\alpha) = e^{-|\alpha|a}$ where $a > 0$.
- 14) Using finite Fourier transformation, solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ given that
 $u(0, t) = 0, u(4, t) = 0$ and $u(x, 0) = 2x$ where $0 < x < 4, t > 0$.
- 15) Given that $F_c[e^{-ax}] = \frac{\sqrt{2}}{\sqrt{\pi}} \frac{a}{a^2 + s^2}$ then find $F_s[e^{-ax}]$.

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Set-2

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V SEM(CBCS) MODEL QUESTION PAPER – B. Sc. MATHEMATICS

PAPER – 5.2a: Analysis and Integral Transforms

Duration : 3 Hours

Max. Marks : 90

PART -A

I Answer any Six Questions

6× 2=12


- 1) Define Gamma and Beta function.
- 2) Evaluate $\beta\left(\frac{9}{2}, \frac{7}{2}\right)$.
- 3) Obtain Fourier Coefficient a_n for the function $f(x) = \begin{cases} -\pi & \text{in } -\pi < x < 0 \\ x & \text{in } 0 < x < \pi \end{cases}$
- 4) Find $L[\cosh at \sin 2t]$
- 5) Find $L[t^2 \sin t]$.
- 6) Write the cosine and sine Fourier integral transformation of $f(x)$.
- 7) Find the Fourier integral expansion of $f(x) = e^{-ax}$ where $x > 0$ and $f(-x) = f(x)$.

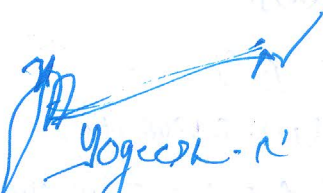
PART -B


II Answer any six questions

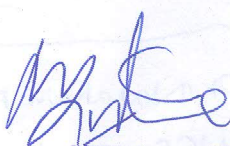
6× 3=18

- 1) Evaluate $\int_0^1 x^{9/2} (1-x)^{-1/2} dx$.
- 2) Prove that $\Gamma(1/2) = \sqrt{\pi}$.
- 3) Obtain the Fourier series of $f(x) = x$ in $(-\pi, \pi)$.
- 4) IF $L[f(t)] = f(s)$ then Prove that $L(f(at)) = \frac{1}{a} f\left(\frac{s}{a}\right)$
- 5) Find $L^{-1}\left[\frac{3s+1}{(s+1)^4}\right]$.
- 6) Show that $e^{-ax} = \frac{2a}{\pi} \int_0^\infty \frac{\cos ux}{u^2 + a^2} du$ $a > 0, x > 0$.
- 7) Prove that $F_s[f'(x)] = -\alpha F_c[f(x)]$.


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PART - C

III Answer any Four Questions.

4 × 5 = 20

- 1) Prove that $\beta(m, n) = \frac{\Gamma(m) \cdot \Gamma(n)}{\Gamma(m+n)}$ where $m, n > 0$.
- 2) Prove that $\Gamma(m) \cdot \Gamma(m + 1/2) = \frac{\sqrt{\pi}}{2^{2m-1}} \Gamma(2m)$.
- 3) Obtain the Fourier Series of the function $f(x) = \begin{cases} 1 & \text{in } -\pi/2 \leq x \leq \pi/2 \\ -1 & \text{in } \pi/2 \leq x \leq 3\pi/2 \end{cases}$
- 4) Obtain the Fourier Series of the function $f(x) = x^2$ in $(-\pi, \pi)$ and deduce that $\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}$
- 5) Obtain the half range cosine series of $f(x) = \begin{cases} x & \text{in } 0 \leq x \leq \pi/2 \\ \pi - x & \text{in } \pi/2 \leq x \leq \pi \end{cases}$

IV Answer any Four Questions

4 × 5 = 20

- 6) Find i) $L[te^{3t} \sin 2t]$ ii) $L[\sin^2 3t]$
- 7) Find $L\left[\frac{2 \sin 2t \cos 3t}{t}\right]$.
- 8) Find $L^{-1}\left[\frac{3s+2}{2s^2-4s+3}\right]$
- 9) Verify the Convolution theorem for the function $f(t) = 1, g(t) = \sin t$
- 10) Solve $\frac{d^2 y}{dt^2} - \frac{dy}{dt} = 5 \sin 2t$ using Laplace transforms
Where $y(0) = y'(0) = 1$.

V Answer any Four Questions

4 × 5 = 20

- 11) Express $f(x) = \begin{cases} 1 & \text{in } |x| \leq 1 \\ 0 & \text{in } |x| > 1 \end{cases}$ as a Fourier integral and hence evaluate $\int_0^{\infty} \frac{\sin u \cos ux}{u} du$
- 12) Find the Fourier Transform of $f(x) = \begin{cases} 1 - x^2 & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| > 1 \end{cases}$
- 13) Find the inverse Fourier transform of $f(\alpha) = e^{-\alpha^2}$.
- 14) Find the Fourier Cosine transform of $f(x) = \left(1 - \frac{x}{\pi}\right)^2$
- 15) Prove that a) $F_c[f'(x)] = -\frac{\sqrt{2}}{\sqrt{\pi}} f(0) + \alpha F_s[f(x)]$.
b) $F_s[f'(x)] = -\alpha F_c[f(x)]$.

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