Set-1

TUMKUR UNIVERSITY

V SEM(CBCS) MODEL QUESTION PAPER – B. Sc. MATHEMATICS
PAPER: 5.1: ADVANCED ALGEBRA AND NUMERICAL METHODS

Time : 3 Hours **Max. Marks : 90**

PART-A

I. ANSWER ANY SIX QUESTIONS. **6X2=12**

1. Add and Multiply the two polynomials over the Ring (Z_6 ,$+_6$, x_6), where $f(x)=2x^0+5x+3x^2$ and $g(x)=1x^0+4x+2x^3$.
2. Define Irreducible element ?
3. Give an example to show that the union of two sub rings is not necessarily a Sub-ring .
4. Express the vector (3,5,2) as the linear combination of the vectors (1,1,0), (2,3,0) and (0,0,1) of $V_3(R)$.
5. Find the Linear transformation of f: $R^2 \rightarrow R^2$ such that f(1,0)=(1,1) and f(0,1)=(-1,2).
6. Explain Newton-Raphson Method to solve f(x)=0.
7. Solve $\frac{dy}{dx} = x + y$ by Euler's Method, given that y=1 when x=0, h=0.2.

PART-B

II. ANSWER ANY SIX QUESTIONS. **6X3=18.**

1. Show that the set M of all 2x2 matrices of the form $\begin{bmatrix} 0 & a \\ 0 & b \end{bmatrix}$ where a, b are integers, is a left ideal but not a right ideal in the right of 2x2 matrices with elements as Integers.
2. If R is a commutative Ring, then
   (i) $a|b$ and $b|c \Rightarrow a|b+c$.
   (ii) $a|b \Rightarrow a|bx$ for all $x \in R$.
3. Let f: $R \rightarrow R^1$ be a homomorphism of rings from R onto R^1 with kernel K, then show that f is one-one iff $K = \{0\}$.
4. In any Vector Space V over a field F. Prove that
   (i) $c.0=0$, $\forall c \in F$ (ii) $0.\alpha = 0$, $\forall \alpha \in F$
   (iii) $(-c)\alpha = -(c\alpha) = c(-\alpha)$, $\forall c \in F$ & $\forall \alpha \in F$
5. For the Matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, find the corresponding Linear Transformation T: $R^2 \rightarrow R^2$ w.r.t the Basis {(1,0), (1,1)}.
6. Use Gauss-Seidel Method to solve the system $x + 5y = 11; 4x + y = 6$ correct to 2 decimal places.
7. Using Picard's method, solve the differential equation $\frac{dy}{dx} = x^2 + y$, $y(0) = 1$ to find the value of y at $x = 0.1$.

(Dr. Narahari N.)
UCS, Tumakuru

Yogeesh N
GFGC Tumkur

M.S. SURESHA
G.F.G.C. TUMKUR

DEAN
Faculty of Science & Technology
Tumkur University
Tumkur-572103

PART -C

III. ANSWER ANY FOUR QUESTIONS 4x5=20.

1. Prove that the Ring (Z_n , $+_n$, x_n) is an Integral Domain and hence a Field if and only if n is a prime.
2. The necessary and sufficient conditions for a non-empty subset K of a field F to be subfield of F are (i) $a \in K,\ b \in K \Rightarrow a-b \in K$.
   (ii) $a \in K,\ 0 \neq b \in K \Rightarrow ab^{-1} \in K$.
3. Show that the union of two ideals of a ring R is an ideal of R if and only if one is contained in the other.
4. Let $f: R \to R^1$ be a homomorphism form the Ring R into R^1 then prove that
   (i) $f(0) = 0^1$ are the zeros of R and R^1 respectively. (ii) $f(-a) = -f(a)$, $\forall\ a \in R$.
5. State and Prove the Fundamental theorem of homomorphism.

IV. ANSWER ANY FOUR QUESTIONS. 4x5=20.

6. Prove that the union of two subspaces of a vector space V over a field F is a subspace if and only if one is contained in the other.
7. In a n-dimensional vector space V(F), (i) any (n+1) elements of V are Linearly Dependent. (ii) no set (n-1) elements can span V.
8. If T is a mapping from $V_2(R)$ into $V_3(R)$ defined by $T(x,y) = (x \cos\theta - y \sin\theta\ ,\ x \sin\theta + y \cos\theta)$, then show that T is a Linear Transformation.
9. Let T: $V \to W$ be a linear transformation and V be a finite dimensional vector space , then prove that $r(T) + n(T) = d(V)$.
10. Show that the linear map $T:V_3 \to V_3$ defined by $T(e_1) = e_1+e_2$, $T(e_2) = e_2+e_3$ and $T(e_3) = e_1+e_2+e_3$ is non-singular and find its inverse.

V. ANSWER ANY FOUR QUESTIONS. 4x5=20.

11. Find the root between 2 and 3 of the equation $x^4-x^3-2x^2-6x-4=0$ up to six iterations correct to two decimal places by using Bisection Method.
12. The equation $x^3-4x+1=0$ has one root between 0 and 1. Find the real root correct to 3 places of decimal by the Method of False Position.
13. Solve by Jacobi's Iteration Method $5x + 2y + z = 12, x + 4y + 2z = 15, x + 2y + 5z = 20$.
14. Using Euler's Modified Method, Solve $\frac{dy}{dx} = x^2 + y$ when y(0)=1 for x = 0.25.
15. Solve $\frac{dy}{dx} = x + y^2$ with the initial condition y =1 when x = 0 for x = 0.2 by Runge - Kutta Method.

(Dr. Narahari N.)
UCS, Tumakuru

Yogeesh N
GFGC Tumkur

M.S. SURESHA
G.F.G.C TUMKUR

DEAN
Faculty of Science & Technology
Tumkur University
Tumkur-572103

Set-2

TUMKUR UNIVERSITY

V SEM(CBCS) MODEL QUESTION PAPER – B. Sc. MATHEMATICS

PAPER: 5.1: ADVANCED ALGEBRA AND NUMERICAL METHODS

Duration: 3 hrs | Max. Marks: 90

PART-A

I. ANSWER ANY 6 QUESTIONS 6X2=12

1. Give an example of the following: (i) A ring with zero divisors (ii) Field.
2. Define an ideal of a ring.
3. State the fundamental theorem of homomorphism of rings.
4. Define a subspace of a vector space. Give an example.
5. Find the rank of the linear transformation whose matrix is $M=\begin{bmatrix}1 & -1 & 2\\ 3 & 1 & 1\\ 2 & 2 & -1\end{bmatrix}$.
6. State the intermediate value theorem.
7. Using Picard's method, solve the differential equation $\frac{dy}{dx}=x+y,\ y(0)=1$ to find the value of y at $x=0.2$ correct to two decimal places.

PART-B

II. ANSWER ANY 6 QUESTIONS 6X3=18

1. Prove that in a ring $(R,+,\cdot)$, $a.0=0.a=0\ \forall a\in R$ where 0 is the additive identity of R.
2. Prove that $S=\left\{\begin{bmatrix}a & 0\\ 0 & 0\end{bmatrix}: a\in Z\right\}$ is a sub-ring of the ring of integers $(Z,+,\cdot)$.
3. Define the following w. r. to a ring: (i) Unit elements (ii) Irreducible elements.
4. In a vector space $(V,+,\cdot)$ over the field F, show that $c.(-\alpha)=(-c).\alpha=-(c\alpha)\ \forall c\in F,\alpha\in V$
5. Define basis and dimension of a vector space.
6. Apply Regula-Falsi method to find a root of the equation $x^2-3x-1=0$ in the interval [3, 4] in two steps.
7. Use the method of Gaussian elimination to solve the system $x-y=5,\ 2x+y=4$.

(Dr. Narahari N)
UCS, Tumakuru

Yogesh L.N
GFGC Tumkur

(M.S. SURESHA)
A-F-G-C TUMKUR

DEAN
Faculty of Science & Technology
Tumkur University
Tumkur-572103

PART-C

III. ANSWER ANY 4 QUESTIONS **4X5=20**

1. Show that the set Z[i] is a Ring under usual addition and multiplication of complex numbers.
2. Prove that the ring $(Z_n, +_n, \cdot_n)$ is an integral domain if and only if n is prime.
3. Show that the intersection of any two subrings of a ring R is again a subring of R. Further, give an example to show that the union of two subrings need not be a subring.
4. Prove that the ring of integers $(Z,+,\cdot)$ is a principal ideal ring.
5. a) Define the kernel of a ring homomorphism.
   b) Prove that $f:(Z,+,\cdot)\to(3Z,+,\cdot)$ defined by $f(x)=3x\ \forall x\in Z$ is an isomorphism.

IV. ANSWER ANY 4 QUESTIONS **4X5=20**

6. Show that the set $V_2(R)=\{(x,y)\mid x,y\in R\}$ forms a vector space over the field R w. r. to component-wise addition and scalar multiplication.
7. Show that the vector $(2,-5,7)\in V_3(R)$ is not in L[S], where $S=\{(1,-3,2),(2,-4,-1),(1,-5,7)\}$.
8. Define linear independence and dependence of a set of vectors. Further, prove that, in a vector space of dimension n, any set of vectors $\{v_1,v_2,\ldots,v_n,v_{n+1}\}$ is linearly dependent.
9. Show that $T:V_2(R)\to V_2(R)$ defined by $T(x,y)=(x+y,x-2y)$ is a linear transformation. Also, find the matrix of the transformation w. r. to the standard ordered basis of $V_2(R)$.
10. Define range and kernel of a linear transformation. Further, if $T:U\to V$ is a linear transformation, prove that range(T) is a subspace of V.

V. ANSWER ANY 4 QUESTIONS **4X5=20**

11. Use the method of bisection to obtain a real root of the equation $xe^x=\cos x$ correct to two decimal places.
12. Obtain a root of the equation $x\cos x-\sin x=0$ near $x=0.5$ using Newton Raphson method.
13. Solve the system of equations $x+10y+z=24,\ x+y+10z=33,\ 10x+y+z=15$ using Gauss- Seidel method taking 4 iterations.
14. Using Euler's modified method, solve $\frac{dy}{dx}+y=1,\ y(0)=0$ for $x=0.2$.
15. Find the value of y at $x=1.25$ given that $\frac{dy}{dx}=xy,\ y(1)=2$ using Runge Kutta method of fourth order.

(Dr. Narahari N.)
UCS, Tumakuru

Yogeesh N
GFGC Tumkur

M.S. SURESHA
R.F.G.C TUMKUR

DEAN
Faculty of Science & Technology
Tumkur University
Tumkur-572103

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V SEM(CBCS) MODEL QUESTION PAPER – B. Sc. MATHEMATICS

PAPER – 5.2a: Analysis and Integral Transforms

Duration : 3 Hours **Max. Marks : 90**

PART- A

I Answer any Six Questions. $6\times 2 = 12$

1) Prove that $\int_0^\infty e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$

2) Prove that $\Gamma(1/4)\Gamma(3/4) = \pi\sqrt{2}$.

3) If $f(x) = \begin{cases} 0 & for \quad -2 < x < 0 \\ 1 & for \quad 0 < x < 2 \end{cases}$ then find b_n.

4) Find $L[sin3t\ cos2t]$

5) Find $L\left[\frac{e^{-t}sint}{t}\right]$

6) Prove that $F[af(t) + bg(t)] = aF[f(t)] + bF[g(t)]$.

7) Write down the Fourier Cosine and Sine Transforms of $f(x)$.

PART -B

II Answer any Six Questions $6\times 3 = 18$

1) Show that $\int_0^a x^4\sqrt{a^2 - x^2}dx = \frac{\pi a^6}{32}$

2) Evaluate $\int_0^\infty \frac{x^4(1+x^5)}{(1+x)^{15}}dx$

3) Obtain the Fourier series of $f(x) = \frac{1}{2}(\pi - x)$ in $-\pi < x < \pi$

4) Find the $L[cosht\ sin^3 2t]$

5) Find $L^{-1}\left[\frac{2s-1}{s^2-2s+10}\right]$

6) Find the Fourier Cosine Transforms of $f(x) = \begin{cases} 1 & for\ 0 \le x < a \\ 0 & for \quad x \ge a \end{cases}$

7) Find the Fourier Integral Expansion of $f(x) = \begin{cases} \pi/2 & for\ |x| < 1 \\ 0 & for\ |x| > 1 \end{cases}$

(Dr. Narahari N.)
UCS, Tumakuru

Yogeesh N
GFGC Tumkur

MSS
(M.S. SURESHA)
G.F.G.C. TUMKUR

Faculty of Science
Tumkur University
Tumkur-572103

PART- C

III Answer any Four Questions $4\times 5 = 20$

1) Prove that $\int_0^{\pi/2} \sin^m\theta \cos^n\theta \, d\theta = \frac{1}{2}\left(\frac{\Gamma(m+1/2)\Gamma(n+1/2)}{\Gamma(m+n+1/2)}\right)$
2) Show that $\int_0^1 \frac{dx}{(1-x^n)^{1/n}} = \frac{\pi}{n} cosec\left(\frac{\pi}{n}\right)$
3) Obtain the Fourier Series of $f(x) = \begin{cases} -\pi \text{ in } & -\pi < x < 0 \\ x \text{ in } & 0 < x < \pi \end{cases}$

   and Hence find that $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$
4) Obtain the Fourier Series of $f(x) = \begin{cases} 1 + \frac{4x}{3} & for \ -3/2 < x \le a \\ 1 - \frac{4x}{3} & for \quad 0 \le x < 3/2 \end{cases}$
5) Obtain half range cosine series of $f(x) = x(\pi - x)$ over the interval $(0, \pi)$

   and hence deduce that $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$

IV Answer any Four Questions: $4 \times 5 = 20$

6) Evaluate (a) $L[e^{-3t}(2cos5t - 3sin5t)]$ (b) $L(1+t)^3$
7) Evaluate (a) $L\int_0^t t\, e^{-t} sinat\, dt$ (b) $L\left[\frac{e^{-t} sint}{t}\right]$
8) Find the $L^{-1}\left[\frac{s}{(s-3)(s^2+4)}\right]$
9) Verify the Convolution theorem for $f(x) = e^t, g(t) = cost$
10) Solve by using Laplace Transforms $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y = e^{2t}$ given that $= 1, \frac{dy}{dt} = -1$ $when\ t = 0$.

V Answer any Four Questions $4 \times 5 = 20$

11) Find the Fourier integral Expansion of $f(x) = \begin{cases} 0 \text{ in } & x < 2 \\ 1 \text{ in } & 2 < x < 3 \\ 0 \text{ in } & x > 3 \end{cases}$
12) Find the Fourier Transform of $f(x) = \begin{cases} 1 - |x| & for \quad |x| \le 1 \\ 0 & for \quad |x| > 1 \end{cases}$ and hence Show

    that $\int_0^\infty \frac{sin^2 t}{t^2} dt = \frac{\pi}{2}$
13) Find the inverse Fourier transform of $f(\alpha) = e^{-|\alpha| a}$ $where\ a > 0.$
14) Using finite Fourier transformation , solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ given that

    $u(0,t) = 0,\ u(4,t) = 0\ and\ u(x,0) = 2x\ where\ 0 < x < 4,\ t > 0.$
15) Given that $F_c[e^{-ax}] = \frac{\sqrt{2}}{\sqrt{\pi}} \frac{a}{a^2+s^2}$ then find $F_s[e^{-ax}]$.

(Dr. Narahari N.)
UCS, Tumakuru

Yogeesh. N
GFGC Tumkur

(M. S. SURESHA)
G.F.G.C TUMKUR

DEAN
Faculty of Science & Technology
Tumkur University
Tumkur-572103

Set-2

TUMKUR UNIVERSITY

V SEM(CBCS) MODEL QUESTION PAPER – B. Sc. MATHEMATICS

PAPER – 5.2a: Analysis and Integral Transforms

Duration : 3 Hours **Max. Marks : 90**

PART -A

I Answer any Six Questions **6× 2=12**

1) Define Gamma and Beta function.
2) Evaluate $\beta\left(\frac{9}{2},\frac{7}{2}\right)$.
3) Obtain Fourier Coefficient a_n for the function $f(x)=\begin{cases}-\pi & in\ -\pi<x<0\\ x & in\ \ 0<x<\pi\end{cases}$
4) Find $L[coshat\ sin2t]$
5) Find $L[t^2 sint]$.
6) Write the *cosine* and *sine* Fourier integral transformation of $f(x)$.
7) Find the Fourier integral expansion of $f(x)=e^{-ax}$ where $x>0$ and $f(-x)=f(x)$.

PART -B

II Answer any six questions **6× 3=18**

1) Evaluate $\int_0^1 x^{9/2}(1-x)^{-1/2}\,dx$.
2) Prove that $\Gamma(1/2)=\sqrt{\pi}$.
3) Obtain the Fourier series of $f(x)=x$ in $(-\pi,\pi)$.
4) IF $L[f(t)]=f(s)$ then Prove that $L(f(at))=\frac{1}{a}f\left(\frac{s}{a}\right)$
5) Find $L^{-1}\left[\frac{3s+1}{(s+1)^4}\right]$.
6) Show that $e^{-ax}=\frac{2a}{\pi}\int_0^\infty \frac{cosux}{u^2+a^2}du\ \ a>0, x>0$.
7) Prove that $F_s[f'(x)]=-\alpha F_c[f(x)]$.

(Dr. Narahari N.)
UCS, Tumakuru

Yogeesh N
GFGC Tumkur

(M. S. SURESHA)
G.F.G.C. TUMKUR.

DEAN
Faculty of Science & Technology
Tumkur University
Tumkur-572103

PART -C

III Answer any Four Questions. **4× 5=20**

1) Prove that $\beta(m,n) = \frac{\Gamma(m).\Gamma(n)}{\Gamma(m+n)}$ *where* m, n > 0.

2) Prove that $\Gamma(m).\Gamma\left(m + {}^{1}/_{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$.

3) Obtain the Fourier Series of the function $f(x) = \begin{cases} 1 \; in -{}^{\pi}/_{2} \le x \le {}^{\pi}/_{2} \\ -1 \; in \; {}^{\pi}/_{2} \le x \le {}^{3\pi}/_{2} \end{cases}$

4) Obtain the Fourier Series of the function $f(x) = x^2$ in $(-\pi, \pi)$ and deduce that $\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}$

5) Obtain the half range cosine series of $f(x) = \begin{cases} x & in \;\; 0 \le x \le {}^{\pi}/_{2} \\ \pi - x & in \;\; {}^{\pi}/_{2} \le x \le \pi \end{cases}$

IV Answer any Four Questions **4× 5=20**

6) Find i) $L[te^{3t}sin2t]$ ii) $L[sin^2 3t]$

7) Find $L\left[\frac{2\, sin2t\, cos3t}{t}\right]$.

8) Find $L^{-1}\left[\frac{3s+2}{2s^2-4s+3}\right]$

9) Verify the Convolution theorem for the function $f(t) = 1, g(t) = sint$

10) Solve $\frac{d^2y}{dt^2} - \frac{dy}{dt} = 5sin2t$ using Laplace transforms
Where $y(0) = y'(0) = 1$.

V Answer any Four Questions **4× 5=20**

11) Express $f(x) = \begin{cases} 1 & in \;\; |x| \le 1 \\ 0 & in \;\; |x| > 1 \end{cases}$ as a Fourier integral and hence evaluate $\int_0^{\infty} \frac{sinu\; cosux}{u} du$

12) Find the Fourier Transform of $f(x) = \begin{cases} 1 - x^2 & for\; |x| \le 1 \\ 0 & for\; |x| > 1 \end{cases}$

13) Find the inverse Fourier transform of $f(\alpha) = e^{-\alpha^2}$.

14) Find the Fourier Cosine transform of $f(x) = \left(1 - \frac{x}{\pi}\right)^2$

15) Prove that a) $F_c[f'(x)] = -\frac{\sqrt{2}}{\sqrt{\pi}} f(0) + \alpha F_s[f(x)]$.
b) $F_s[f'(x)] = -\alpha F_c[f(x)]$.

(Dr. Narahari N.)
UCS, Tumkur

Yogeesh N
GFGC Tumkur

(M.S. SURESHA)
A.F.R.C. TUMKUR.

DEAN
Faculty of Science & Technology
Tumkur University
Tumkur-572103